One of These Things is Not Like the Other: Two Questions in Algebraic Group Theory

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I'll admit, I'm breaking rules.

Ethics in Mathematics

Conjugacy Classes of Unipotent Elements in Generalized
Symmetric Spaces over Finite Fields

Ethics (or the lack thereof) in Mathematics

Mathematics is not neutral and mathematics was never neutral.
Mathematics, as it is studied and shared, is a reflection of the values of the society and sometimes the values of the mathematicians.

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Mathematics Education in Nazi Germany

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- Calculus problems about gassing populations and bomb trajectories.
- Skull Index, Face Index, and Profile Angle having the 'German Standard'.
- Population percentages and asking "What danger do you perceive for the future of the German people?"

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Careful estimates place the number of mentally ill patients, epileptics and others in (German) institutions at 300,000 persons. What is the total yearly cost of this at four Reich marks per person per day? How many new Marriage Grant loans at 1000 marks each could this sum provide?

Note: Loans required demonstrating ancestral and medical "fitness" and women had to leave work. This made more room for unemployed men.

Current Mathematics Curriculum:

Topics we see:
- money and banking
- pop-culture and TV
- health and food
- shopping and consumerism
- science-related

Topics we don't see:
- distribution of resources (wealth, income, food, etc.)
- school-to-prison pipeline, mass incarceration
- racism
- sexism
- environmental issues
- socioeconomic status

Ethics in the Practice of Mathematics-Examples

Cambridge Analytica: how did it turn clicks into votes?

Black-Scholes: The maths formula linked to the financial crash

Researchers use facial recognition tools to predict sexual orientation. LGBT groups aren't happy.

The actual patented algorithm used by PredPol is displayed below:

$$\frac{\partial A}{\partial t} = B + \frac{\eta D}{4} \nabla^2 A - \omega A + \theta \omega \delta$$

Can you spot a terrorist just by looking at their face? New software can tell if you are anything from a paedophile to an ace poker player by analysing your features

Court: Public Deserves to Know How NYPD Uses Predictive Policing Software

LAPD ends another data-driven crime program touted to target violent offenders

Britain's electronic eavesdropping agency **GCHQ was also accused of gathering information** on the online companies via Prism.

China wants to track and grade each citizen's actions — it's in the testing phase

US spy agency 'collects phone records'

Ethics in the Practice of Mathematics-Examples

Plagarism and self-plagarism vs. "folk theorems"

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Mathematics usage as a filtration device

Mathematical elitism and the myth of the "math genius"

One of these things is not like the other?

GeoPhysics, Statistics, Computer Science, Conservation Biology, general science...have professional societies with strong Ethical Codes/Codes of Conduct that address the responsibilities in the practice.

Both of the US-based math societies (AMS, MAA) have statements of ethics largely surrounding issues of plagiarism and publication (as of last check, the third (SIAM) does not).

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Why does this matter?

We are training the next generation of mathematicians, data scientists, computer scientists, and statisticians. Articulating guidelines and standards supports those in the profession and practice informs our teaching.

Current Work in Ethics

The existence theorem: Ethics exist in mathematics.

The construction theorem: What are the principles from which we build our ethical practice?

Conjugacy Classes of Elements in Generalized Symmetric Spaces over Finite Fields

Two Questions

Ethics
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Algebraic Group Theory
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Joy
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Conjugacy Classes of Elements in Generalized Symmetric Spaces over Finite Fields

Conjugacy Classes – 'conjugation' is specific action defined by multiplying on the left by an element and on the right by the inverse of an element. Classes are the sets of objects that can be reached under the action.

- Action: Features of the Paint App
- Classes: The number of sets were left after we used our tools to reduce them.

Conjugacy Classes of Elements in Generalized Symmetric Spaces over Finite Fields

Generalized Symmetric Spaces – a special class of matrices that satisfy a particular property.

- Space: All Beyonce pictures

Conjugacy Classes of Elements in Generalized Symmetric Spaces over Finite Fields

Finite Fields – fields are groups of numbers or objects that satisfy certain axioms (the real numbers are an example). Finite fields means there are finitely many objects (the real numbers are infinite)

- Finite Field: Just pictures of Beyonce outfits from Coachella

Addition and Multiplication Tables for {0,1,2,3,4,5,6,7,8,9} in the real numbers

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	56	63	72	81

Addition and Multiplication Tables for {0,1,2,3,4} in the finite field of five elements

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

X	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

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$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 4 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 4 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Conjugation of matrices in H

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Conjugation of matrices in H

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Thank you!

While you think of questions, please enjoy some pictures taken by
Ian Murray.

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